

# Origin of causal set structure in the quantum universe

Jon Eakins and George Jaroszkiewicz

School of Mathematical Sciences, University of Nottingham,  
University Park, Nottingham NG7 2RD, UK

February 7, 2008

## Abstract

We discuss the origin of causal set structure and the emergence of classical space and time in the universe. Given that the universe is a closed self-referential quantum automaton with a quantum register consisting of a vast number of elementary quantum subregisters, we find two distinct but intimately related causal sets. One of these is associated with the factorization and entanglement properties of states of the universe and encodes phenomena such as quantum correlations and violations of Bell-type inequalities. The concepts of separations and entanglements of states are used to show how state reduction dynamics generates the familial relationships which gives this causal set structure. The other causal set structure is generated by the factorization properties of the observables (the Hermitian operators) over the quantum register. The concept of skeleton sets of operators is used to show how the factorization properties of these operators could generate the classical causal set structures associated with Einstein locality.

PACS number(s): 03.65.Ca, 03.65.Ud, 04.20.Gz, 98.80.Qc

# I. INTRODUCTION

Whilst attempting to account for the existence of space, time and matter in the universe, physicists often adopt one of two opposing viewpoints. These may be labelled *bottom-up* and *top-down* respectively, reflecting the basic difference between reductionist and holistic physics. Such a difference is to be expected given a universe running on quantum principles, because quantum mechanics ( $QM$ ) is simultaneously reductionist and holistic.

Many bottom-up approaches proceed from the assertion that at its most basic level, the universe can be represented by a vast collection of discrete events embedded in some sort of mathematical space, such as a manifold. By avoiding any assumption of a pre-existing manifold, the *pre-geometric* approach goes further and asserts that conventional classical time and space and the classical reality that we appear to experience all emerge on macroscopic scales due to the complex connections between these fundamental microscopic entities. This approach was pioneered by Wheeler [1] and has received attention more recently by Stuckey [2].

On the other hand, most top-down approaches to quantum cosmology consider the universe as a single quantum system with a unique state evolving according to a given set of laws or conditions. From this point of view, our classical picture of the universe is no more than a good working approximation whenever a detailed quantum mechanical description can be neglected.

In this paper we apply a theoretical framework or paradigm which attempts to explain and reconcile these viewpoints within a consistent set of postulates based on the principles of standard quantum mechanics, stretched to cover the unique case when the system under observation is the universe and not just a subsystem of it. In this paradigm, reviewed briefly in the Appendix, discrete structures occur naturally, being generated by the factorization and entanglement properties of states and observables.

Related to our approach is the bottom-up approach to cosmology known as the causal set hypothesis [3, 4, 5, 6, 7]. This assumes that spacetime is discrete at the most fundamental level. In those models it is postulated that classical, discrete events are generated at random, though neither the nature of these events nor the mechanism generating them is explained or discussed in detail. In this paper we explain how the particular mathematical properties and dynamical principles of quantum theory can generate such a structure. A particularly important feature of our approach is the occurrence of two distinct causal set structures, in contrast to the one normally postulated. One of these causal sets arises from the entanglement and separation properties of states whilst the other arises from the separation and entanglement properties of the operators. We believe that these two causal sets

correspond to the two sorts of information transmission observed in physics, that is, non-local quantum correlations which do not respect Einstein locality and classical information transmissions which do respect it.

Although it seems natural to generate causal sets by the discretization of a pseudo-Riemannian spacetime manifold of fixed dimension, as is done in lattice gauge theories and the Regge discretization approach [8] to general relativity (*GR*), causal set events need not in principle be regarded as embedded in some background space of fixed dimension  $d$ . Instead, conventional (i.e. physical) spacetime is expected to emerge in some appropriate limit as a consequence of the causal set relations between the discrete events. It is expected that in the correct continuum limit, metric structure should emerge [4]. Additionally, it has been suggested [3] that the dimension of this emergent spacetime might be a scale dependent quantity, making the model potentially compatible with general relativity in four spacetime dimensions, string and m-brane cosmology and higher dimensional Kaluza-Klein theories. The attractive feature here is the idea of some universal “pregeometric” structure capable of accounting for all known emergent features and from which most, if not all, currently popular models of the universe would emerge as reasonable approximations in the right contexts.

Important contemporary issues in quantum mechanics which impact directly on quantum cosmology and hence on our work are the status of state reduction [9] and the physical meaning of time. For a number of reasons, state reduction has often been regarded as an unattractive modification of more elegant quantum principles based on Schrödinger evolution. In the many worlds and decoherence interpretations of quantum mechanics for instance, quantum jumps are regarded in principle as non-existent, at best being useful approximations to a underlying continuous quantum evolution. The physical meaning of time is intimately bound up with this issue, because in the physics laboratory and in the sphere of more general human activity, physical time is marked by irreversible processes of information acquisition and loss, the origin of which is normally attributed to state reduction.

Up to relatively recently, it was reasonable to follow Schrödinger in his view that quantum mechanics is a theory dealing with ensembles, with the consequence that wavefunctions should be regarded as no more than an encoding of statistical information. Certainly, there has been in recent years a view amongst many leading theorists that quantum states should not be regarded as real attributes of systems.

This impacts on the interpretation of time. Cutting a long story short, there are two distinct views of time, which we will refer to as the manifold time and process time perspectives respectively. The former considers time as part of a four dimensional continuum for which relativistic principles hold.

Physical theories have to be Lorentz covariant in their predictions of local expectation values and there is no inherent difference between past and future on the most superficial levels of the theory. This leads to the block universe picture of a universe in which structural (i.e., geometrical) properties hold predominantly.

The process view of time relates to dynamical processes of change. According to this view, only the moment of the present has physical meaning and the future can only be discussed in terms of potential. It is well understood that this perspective does not square with relativistic principles, because in relativity there is no absolute simultaneity. However, both perspectives are consistent with quantum principles. In the manifold perspective, we can use quantum mechanics to calculate expectation values for past, present and future, if we ignore the thorny issue of exactly what goes on when information is extracted from physical systems (the measurement problem). From the process time perspective, quantum mechanics gives probabilities for individual future outcomes of quantum tests.

Consistent with the notion that single outcomes can be physically relevant in quantum mechanics are the recent developments in quantum physics concerning single ion traps [9]. There, a single subsystem is subject to external probing in such a way that the best description of what is going on is in terms of quantum jumps. It seems no longer possible to maintain Schrödinger's view that we can never experiment on single particles but only on ensembles.

These experimental developments support the case that, contrary to what decoherence asserts, quantum state reduction processes are meaningful. Indeed, state reduction is as equally important in quantum computation as the unitary evolution characterizing the relationship between initial and final states and the decohering influence of the environment.

The present paper incorporates the notion of an underlying qubit pregeometry with state reduction into a specific paradigm, which for convenience we refer to as the stages paradigm. We believe that the mathematical structures occurring naturally in quantum registers have the potential to answer a number of questions important to quantum physics, such as the relationship of the observer to the system under observation. Many directions still have to be explored. In this paper we will lay down general ideas, explaining how a causal set structure can arise naturally within the stages paradigm, and how much of the Hasse diagrams generated in causet theory may be recreated as the state of the universe  $\Psi_n$  factorizes and re-factorizes over successive jumps. We believe, however, that the classically motivated line of thinking in [5] is too general, because whilst it may seem mathematically possible to produce any configuration of elements in a causet, not all types of relation-

ship specific to classical Hasse diagrams are permissible in our approach to quantum physics unless some sort of external ‘information store’ is available. Conversely, evolution of states in the stages paradigm will be shown to reproduce only those parts of the Hasse diagrams that are allowed by quantum mechanics and are hence physically meaningful.

## A. Plan

In §2 we review classical causal set theory, followed by a discussion of some points relevant to quantum cosmology in §3. In §4 we introduce the notions of separations and entanglements, which describe the possible partitions of a direct product Hilbert space into subsets of relevance to physics. These concepts provide a natural basis for causal set structure once dynamics is introduced. This is discussed in §5, where we show how state reduction concepts inherent to the stages paradigm lead to a natural definition of the concepts of families, parents and siblings used in causal set theory. In §6 we discuss how two sorts of causal set structure arise in our paradigm, one associated with states and the other with observables, and explain how these are related to non-local quantum correlations and Einstein locality respectively. In §7, we introduce the notion of *skeleton set*, used to construct observables, and discuss the concepts of separability and entanglement for operators. In §8 we discuss the fundamental role of eigenvalues in the theory, in §9 we discuss some physically motivated examples, followed by our summary and conclusions in §10. In the Appendix we review the stages paradigm, which is central to our work.

## II. CAUSAL SETS

A number of authors [3, 4, 5, 10, 11] have discussed the idea that spacetime could be discussed in terms of causal sets. In the causal set paradigm, the universe is envisaged as a set  $\mathcal{C} \equiv \{x, y, \dots\}$  of objects (or *events*) which may have a particular binary relationship amongst themselves denoted by the symbol  $\prec$ , which may be taken to be a mathematical representation of a temporal ordering. For any two different elements  $x, y$ , if neither of the relations  $x \prec y$  nor  $y \prec x$  holds then  $x$  and  $y$  are said to be *relatively space-like*, *causally independent* or *incomparable* [12]. The objects in  $\mathcal{C}$  are usually assumed to be the ultimate description of spacetime, which in the causal set hypothesis is often postulated to be discrete [5]. Minkowski spacetime is an example of a causal set with a continuum of elements [4], with the possibility of extending the relationship  $\prec$  to include the concept of *null* or *lightlike* relationships.

The causal set paradigm supposes that for given elements  $x, y, z$  of the causal set  $\mathcal{C}$ , the following relations hold:

$$\begin{aligned} \forall x, y, z \in \mathcal{C}, \quad x \prec y \text{ and } y \prec z &\Rightarrow x \prec z && \text{(transitivity)} \\ \forall x, y \in \mathcal{C}, \quad x \prec y &\Rightarrow y \not\prec x && \text{(asymmetry)} \\ \forall x \in \mathcal{C}, \quad x &\not\prec x. && \text{(irreflexivity)} \end{aligned} \tag{1}$$

A causal set may be represented by a Hasse diagram [12]. In a Hasse diagram, the events are shown as spots and the relations as solid lines or links between the events, with emergent time running from bottom to top.

One method of generating a causal set is via a process of ‘sequential growth’ [5]. At each step of the growth process a new element is created at random, and the causal set is developed by considering the relations between this new event and those already in existence. Specifically, the new event  $y$  may either be related to another event  $x$  as  $x \prec y$ , or else  $x$  and  $y$  are said to be unrelated. Thus the ordering of the events in the causal set is as defined by the symbol  $\prec$ , and it is by a succession of these orderings, i.e. the growth of the causet, that constitutes the passage of time. The relation  $x \prec y$  is hence interpreted as the statement: “ $y$  is to the future of  $x$ ”. Further, the set of causal sets that may be constructed from a given number of events can be represented by a Hasse diagram of Hasse diagrams [5].

The importance of causal set theory is that in the large scale limit of very many events, causal sets may yield all the properties of continuous spacetimes, such as metrics, manifold structure and even dimensionality, all of which should be determined by the dynamics [3]. For example, it should be possible to use the causal order of the set to determine the topology of the manifold into which the causet is embedded [4]. This is the converse of the usual procedure of using the properties of the manifold and metric to determine the lightcones of the spacetime, from which the causal order may in turn be inferred.

Distance may be introduced into the analysis of causal sets by considering the length of paths between events [3, 4]. A *maximal chain* is a set  $\{a_1, a_2, \dots, a_n\}$  of elements in a causal set  $\mathcal{C}$  such that, for  $1 \leq i \leq n$ , we have  $a_i \prec a_{i+1}$  and there is no other element  $b$  in  $\mathcal{C}$  such that  $a_i \prec b \prec a_{i+1}$ . We may define the path length of such a chain as  $n - 1$ . The distance  $d(x, y)$  between comparable [12] elements  $x, y$  in  $\mathcal{C}$  may then be defined as the maximum length of path between them, i.e. the ‘longest route’ allowed by the topology of the causet to get from  $x$  to  $y$ . This implements Riemann’s notion that ultimately, distance is a counting process [3]. For incomparable elements, it should be possible to use the binary relation  $\prec$  to provide an analogous definition of distance, in much the same way that light signals

may be used in special relativity to determine distances between spacelike separated events.

In a similar way, “volume” and “area” in the spacetime may be defined in terms of numbers of events within a specified distance. Likewise, it should be possible to give estimates of dimension in terms of average lengths of path in a given volume. An attractive feature of causal sets is the possibility that different spatial dimensions might emerge on different physical scales [3], whereas in conventional theory, higher dimensions generally have to be put in by hand.

Our approach differs from the above in certain important respects. In the stages paradigm, reviewed in the Appendix, spacetime *per se* does not exist and therefore cannot be regarded as being discrete or otherwise. Our discrete sets arise naturally from the separation and entanglement properties of quantum states and operators and are therefore not related directly to discrete space or to the discretization of space.

Furthermore, in the stages paradigm, various relations assumed in “sequential growth” must be interpreted carefully. In quantum physics, past, present and future can never have equivalent status. At best we can only talk about conditional probabilities, such as asking for the probability of a possible future stage *if* we assumed we were in a given present stage. This corresponds directly to the meaning of the Born interpretation of probability in *QM*, where all probabilities are conditional: *if* we were to prepare  $\Psi$ , then the conditional probability of subsequently detecting  $\Phi$  is given by  $P(\Phi|\Psi) \equiv |\langle\Phi|\Psi\rangle|^2$ . This does not mean that we actually have to prepare  $\Psi$ .

Another problem is that, as they stand, the classical causal set relations discussed above suggest that the various elements  $x, y, z$  have an independent existence outside of the relations themselves and that these relations merely reflect some existing attributes. This is a “block universe” perspective [13] which runs counter not only to the process time perspective but also to the basic principles of quantum mechanics, the implications of which lead to the uncertainty principle, violations of Bell inequalities and the Kochen-Specker theorem [14]. All of these support Bohr’s view that the quantum analogues of classical values such as position and momentum do not exist independently of observation.

A further criticism of classical causal sets from the point of view of quantum theory comes from an interpretation of what the Hasse diagrams actually represent. In some diagrams, relatively spacelike events with no previous causal connection are permitted to be the parents of the same event. The problem is, given two such unrelated events at a given time  $n$ , it is not clear how any information from either of them could ever coincide, that is, be

brought together to be used to create any mutual descendants unless there is some external agency organizing the flow of that information. The whole point of causal set theory, however, is that there is no external space in which these events are embedded, or any external “memory”, observer or information store correlating such information, and so it is not clear how such processes could be encoded into the dynamics. In our approach, there are actually two dynamically interlinked causal sets, rather than one, with different but interlinked properties stemming from the underlying Hilbert space structure, and this solves this particular problem.

### III. QUANTUM COSMOLOGY

Causal sets were devised as explanations of how the universe might run classically. The introduction of quantum mechanics into the discussion leads to a picture of the universe as a vast quantum automaton or quantum computation. However, not all authors agree that there exists an explicit wavefunction for the universe. Fink and Leschke [15] argued that the universe cannot be treated as a complete quantum system because by definition, the universe cannot be part of an ensemble, nor can it have any sort of external observer. Other authors [16] have argued that there is no objective meaning to the notion of a quantum state *per se* other than in the context of measurement theory with exo-physical observers.

Our counter arguments are based principally on the lack of evidence for any identifiable “Heisenberg cut”, or dividing line, between classical and quantum perspectives. From the subatomic to galactic scales, when looked at carefully, every part of the universe seems to be described by quantum mechanics. In view of the overwhelming empirical success of quantum mechanics on all scales, the conclusion we draw is that quantum principles must govern everything, including the universe itself. In the particular case of the universe, it must be regarded as a unique system for which the conventional quantum rules concerning observers cannot be applied, because they were only ever formulated with reference to true subsystems of the universe. For all such subsystems, an “exo-physical” quantum mechanical description is possible (where the observer looks into a subsystem “from the outside”), but for the unique case of the universe, an “endo-physical” description (where all observers are part of the system) can be physically meaningful.

An important corollary to this line of thought is that if indeed the universe may be represented by a quantum state, then that state can only be a pure state. Any “state of the universe” cannot be a mixed state, because there is no physical meaning to classical uncertainty in this context. The universe



cannot be part of an ensemble as far as the present is concerned, and the only option therefore is to use a pure state description. This is a central tenet of our approach, discussed in the Appendix.

In the long term, it will be necessary to give a detailed account of how such pure states could be used to discuss conventional quantum physics. That is an aspect of our work which is related to the problem of *emergence*, i.e., an account of how the world that we think we see arises from a more fundamental quantum basis. This is a difficult programme which we cannot comment further on here, save to say that in our approach, we expect the factorization properties of states and operators will give a handle on the issue.

Given that the universe is a complete quantum system, there remains the fundamental issue of state reduction (wave-function collapse) versus Schrödinger evolution, with a sharp split between those who do believe in state reduction and those that do not. The many-worlds paradigm [17] and its developments [18, 19] explicitly rule out state reduction, as do various quantum cosmological theories and the general framework known as decoherence. On the other hand, the standard position stated in *QM* texts [14], assumed here to be the origin of the notion of time as a dynamical process [20], takes at face value the role of information acquisition as the true meaning of time. To quote Wheeler [1], “*The central lesson of the quantum has been stated in the words ‘No elementary phenomenon is a phenomenon until it is an observed (registered) phenomenon’.*”

We take the position that if the universe is described by a deterministic Schrödinger equation, with no probabilistic interpretation attributed to the wave-function, then there is no way that true quantum randomness could ever emerge. Any discussion of quantum randomness in such a paradigm would be a pseudo-randomness based on emergent approximations, such as partial tracing over various degrees of freedom so as to provide some sort of justification for the appearance of mixed states in the theory. It is acknowledged by the practitioners of many-worlds and decoherence that accounting for the Born probability rule presents a serious problem in those paradigms. Moreover, because all the structures in the universe are deterministic in such paradigms, then even such approximations and their apparent random outcomes would be predetermined. We take it as self-evident that genuine randomness cannot occur in any system based entirely on deterministic equations. The only option left within such systems is to introduce ad-hoc elements such as semi-classical observers with free will.

## IV. SPLITS, PARTITIONS, SEPARATIONS AND ENTANGLEMENTS

One of the features of quantum mechanics which distinguishes it from classical mechanics is the occurrence of entangled states. However, as we have stressed in [21], an equally important feature is the existence of separable states, i.e., states which are direct products of more elementary states and therefore are not entangled. Some of the factors of such states are used by physicists to represent subsystems under observation, whilst other factors are often used to represent the environment and pointer states of apparatus. In our paradigm, the possibility of various factors remaining relatively unchanged as the universe jumps should account for the occurrence of large scale structures having a “trans-temporal” identity of sorts (in a statistical sense), long enough for classical descriptions to be applicable to them. Some of these structures would to all intents and purposes behave as semi-classical observers, whilst others would be identified with systems under observation. In our paradigm, there is no inherent difference between the concepts of observer and system, except possibly for the scales associated with them. If the universe is describable by a tensor product Hilbert space consisting of more than  $10^{180}$  qubits, as we estimate [21], then there are sufficiently many degrees of freedom to describe vast numbers of different “observers” and vast numbers of different “systems”.

In our paradigm, separations and entanglements play equally fundamental roles and it becomes necessary to introduce a convenient notation to discuss them, as follows.

By definition, tensor product Hilbert spaces contain both separable and entangled elements. We shall call such a space a quantum register. We base our discussion on a finite dimension quantum register  $\mathcal{H}_{[1\dots N]}$  which is the tensor product

$$\mathcal{H}_{[1\dots N]} \equiv \mathcal{H}_1 \otimes \mathcal{H}_2 \otimes \dots \otimes \mathcal{H}_N \quad (2)$$

of a vast number  $N$  of factor Hilbert spaces  $\mathcal{H}_i$ ,  $1 \leq i \leq N$ , each known as a quantum subregister. The dimension  $d_i$  of the  $i^{th}$  quantum subregister will be assumed to be prime. When this dimension is two, such a subregister is known as a quantum bit, or qubit. We restrict our attention to quantum sub-registers of prime dimension because we shall suppose that any Hilbert space which has a dimension  $d = pq$ , where  $p$  and  $q$  are integers greater than one, is isomorphic to the tensor product of two Hilbert spaces of dimensions  $p$  and  $q$  respectively.

In our usage of the tensor product symbol  $\otimes$ , the ordering is not taken to be significant. Left-right ordering is conventionally used as a form of labelling in differential geometry, for example, but in the context of three

or more subregisters, entanglements can occur between elements of any of the subregisters. On account of this it is better to use subscript labels such as in (2) to identify specific subregisters (which are therefore regarded as having their own identities), rather than left-right position. For instance, if  $(j_1 j_2 \dots j_N)$  is any permutation of  $(1 2 \dots N)$  then we may write (2) in the equally valid form

$$\mathcal{H}_{[1\dots N]} = \mathcal{H}_{j_1} \otimes \mathcal{H}_{j_2} \otimes \dots \otimes \mathcal{H}_{j_N}. \quad (3)$$

There is therefore no natural ordering of quantum sub-registers in our approach, and in the long run this is related to quantum non-locality. In any case, any proposed ordering would have two clear problems. First, it would suggest that any given subregister was “further away” from some subregisters than others, and secondly, there is no obvious criterion for making such an ordering anyway. On account of this lack of intrinsic ordering, it is important to understand that the quantum subregisters are not regarded *a priori* as being embedded in any way in some pre-existing manifold.

We shall call this the non-locality property of our tensor products. This property holds for states as well as sub-registers. For example, if  $|\Psi\rangle \equiv |\psi\rangle_1 \otimes |\phi\rangle_2$  is a separable element of  $\mathcal{H}_{[12]} \equiv \mathcal{H}_1 \otimes \mathcal{H}_2$ , such that  $|\psi\rangle_1 \in \mathcal{H}_1$  and  $|\phi\rangle_2 \in \mathcal{H}_2$ , then we may equally well write  $|\Psi\rangle = |\phi\rangle_2 \otimes |\psi\rangle_1$ .

### Splits

A *split* is any convenient way of grouping the  $N$  subregisters in  $\mathcal{H}_{[1\dots N]}$  into two or more factors, each of which is itself a tensor product of subregisters and therefore a vector space. The nonlocality property of tensor products in general permits many different splits of the same quantum register. For example, we may write the register  $\mathcal{H}_{[123]}$  in 5 different ways:

$$\begin{aligned} \mathcal{H}_{[123]} &= \mathcal{H}_1 \otimes \mathcal{H}_{[23]} = \mathcal{H}_3 \otimes \mathcal{H}_{[12]} = \mathcal{H}_2 \otimes \mathcal{H}_{[13]} \\ &= \mathcal{H}_1 \otimes \mathcal{H}_2 \otimes \mathcal{H}_3. \end{aligned} \quad (4)$$

Splits are important to our quantum causal sets because the number of families in a given transition amplitude equals the number of factors in the corresponding split of the total quantum register. In general, the number of ways of splitting a register with  $n$  subregisters is given by the  $n^{th}$  Bell number  $B_n$ , which satisfies the recursion relations

$$B_{n+1} = \sum_{k=0}^n \binom{n}{k} B_k, \quad B_0 = 1, \quad (5)$$

the solution of which is given explicitly by Dobinski's formula

$$B_n = \frac{1}{e} \sum_{k=0}^{\infty} \frac{k^n}{k!}. \quad (6)$$

This sequence grows rapidly with  $n$ , which indicates that quantum causal sets can have very complex structure.

### Partitions

Now  $\mathcal{H}_{[1\dots N]}$  is a vector space of dimension  $d = d_1 d_2 \dots d_N$  which contains both entangled and separable states, but this classification of all states in  $\mathcal{H}$  into separable or entangled sets is too limited in the context of causal sets and the stages paradigm. Mathematicians generally prefer to work with vector spaces whereas physicists are more concerned with particular subsets of vector spaces, so we must extend our classification of vectors to distinguish separable and entangled vectors. We will explain our terminology starting with the separable sets.

Henceforth, we will assume that each subregister cannot itself be split in any way, i.e., is an elementary subregister, such as a qubit.

Take any two subregisters  $\mathcal{H}_i, \mathcal{H}_j$ , in (2) such that  $1 \leq i, j \leq N$  and  $i \neq j$ . We define the *separation*  $\mathcal{H}_{ij}$  of  $\mathcal{H}_{[ij]} \equiv \mathcal{H}_i \otimes \mathcal{H}_j$  to be the subset of  $\mathcal{H}_{[ij]}$  consisting of all separable elements, i.e.,

$$\mathcal{H}_{ij} \equiv \{|\phi\rangle_i \otimes |\psi\rangle_j : |\phi\rangle_i \in \mathcal{H}_i, |\psi\rangle_j \in \mathcal{H}_j\}. \quad (7)$$

By definition, we include the zero vector  $0_{ij}$  of  $\mathcal{H}_{[ij]}$  in  $\mathcal{H}_{ij}$  because this vector can always be written as a trivially separable state, i.e.,

$$0_{ij} = 0_i \otimes 0_j. \quad (8)$$

The separation  $\mathcal{H}_{ij}$  will be called a *rank-2 separation* and this generalizes to higher rank separations as follows. Pick an integer  $k$  in the interval  $[1, N]$  and then select  $k$  different elements  $i_1, i_2, \dots, i_k$  of this interval. Then the *rank- $k$  separation*  $\mathcal{H}_{i_1 i_2 \dots i_k}$  is the subset of  $\mathcal{H}_{[i_1 \dots i_k]} \equiv \mathcal{H}_{i_1} \otimes \mathcal{H}_{i_2} \otimes \dots \otimes \mathcal{H}_{i_k}$  given by

$$\mathcal{H}_{i_1 i_2 \dots i_k} \equiv \{|\psi_1\rangle_{i_1} \otimes \dots \otimes |\psi_k\rangle_{i_k} : |\psi_a\rangle_{i_a} \in \mathcal{H}_{i_a}, \quad 1 \leq a \leq k\}. \quad (9)$$

Every element of a rank- $k$  separation has  $k$ — factors. A rank-1 separation of a subregister is by definition equal to that subregister and so we may write

$$\mathcal{H}_i = \mathcal{H}_{[i]}. \quad (10)$$

Now we are in a position to construct the *entanglements*, which are defined in terms of complements. Starting with the lowest rank possible, we define the *rank-2 entanglement*  $\mathcal{H}^{ij}$  to be the complement of  $\mathcal{H}_{ij}$  in  $\mathcal{H}_{[ij]}$ , i.e.,

$$\mathcal{H}^{ij} \equiv \mathcal{H}_{[ij]} - \mathcal{H}_{ij} = (\mathcal{H}_{[ij]} \cap \mathcal{H}_{ij})^c. \quad (11)$$

Hence

$$\mathcal{H}_{[ij]} = \mathcal{H}_{ij} \cup \mathcal{H}^{ij}. \quad (12)$$

Note that  $\mathcal{H}_{ij}$  and  $\mathcal{H}^{ij}$  are disjoint and  $\mathcal{H}^{ij}$  does not contain the zero vector. An important aspect of this decomposition is that neither  $\mathcal{H}_{ij}$  nor  $\mathcal{H}^{ij}$  is a vector space, as can be readily proved.

The generalization of the above to larger tensor product spaces is straightforward but first it will be useful to extend our notation to include the concept of *separation product*. If  $\mathcal{H}'_i$  and  $\mathcal{H}'_j$  are arbitrary subsets of  $\mathcal{H}_i$  and  $\mathcal{H}_j$  respectively, where  $i \neq j$ , then we define the separation product  $\mathcal{H}'_i \bullet \mathcal{H}'_j$  to be the subset of  $\mathcal{H}_{[ij]}$  given by

$$\mathcal{H}'_i \bullet \mathcal{H}'_j \equiv \{|\psi\rangle \otimes |\phi\rangle : |\psi\rangle \in \mathcal{H}'_i, |\phi\rangle \in \mathcal{H}'_j\}. \quad (13)$$

This generalizes immediately to any sort of product. For example, we see

$$\mathcal{H}_{ij} = \mathcal{H}_i \bullet \mathcal{H}_j. \quad (14)$$

The separation product is associative, commutative and cumulative, i.e.

$$\begin{aligned} (\mathcal{H}_i \bullet \mathcal{H}_j) \bullet \mathcal{H}_k &= \mathcal{H}_i \bullet (\mathcal{H}_j \bullet \mathcal{H}_k) \equiv \mathcal{H}_{ijk} \\ \mathcal{H}_{ij} \bullet \mathcal{H}_k &= \mathcal{H}_{ijk}, \end{aligned} \quad (15)$$

and so on. The separation product can also be defined to include entanglements. For example,

$$\mathcal{H}^{ij} \bullet \mathcal{H}_k = \{|\phi\rangle_{ij} \otimes |\psi\rangle_k : |\phi\rangle_{ij} \in \mathcal{H}^{ij}, |\psi\rangle_k \in \mathcal{H}_k\} \quad (16)$$

A further notational simplification is to use a single  $\mathcal{H}$  symbol, using the vertical position of indices to indicate separations and entanglements, and incorporating the separated product symbol  $\bullet$  with indices directly. For example, the following are equivalent ways of writing the same separation product of entanglements and separations:

$$\mathcal{H}_{23468}^{15\bullet97} \equiv \mathcal{H}_{28\bullet4\bullet36}^{15\bullet97} \equiv \mathcal{H}^{15} \bullet \mathcal{H}^{97} \bullet \mathcal{H}_{28} \bullet \mathcal{H}_4 \bullet \mathcal{H}_{36}. \quad (17)$$

Associativity of the separation product applies to both separations and entanglements. For example, we may write

$$\mathcal{H}_{ij} \bullet \mathcal{H}_{klm} \bullet \mathcal{H}^{rs} = \mathcal{H}_{ij \bullet klm}^{rs} = \mathcal{H}_{ijklm}^{rs}, \quad (18)$$

but note that whilst

$$\mathcal{H}_{ij} \bullet \mathcal{H}_{kl} = \mathcal{H}_{ij \bullet kl} = \mathcal{H}_{ijkl}, \quad (19)$$

we have

$$\mathcal{H}^{ij} \bullet \mathcal{H}^{kl} \equiv \mathcal{H}^{ij \bullet kl} \neq \mathcal{H}^{ijkl}. \quad (20)$$

In practice, rank-3 and higher entanglements such as  $\mathcal{H}^{ijkl}$  have to be defined in terms of complements, in the same way that  $\mathcal{H}^{ij}$  is defined. For example, consider  $\mathcal{H}_{[abc]} \equiv \mathcal{H}_a \otimes \mathcal{H}_b \otimes \mathcal{H}_c$ . We define

$$\mathcal{H}^{abc} \equiv \mathcal{H}_{[abc]} - \mathcal{H}_{abc} \cup \mathcal{H}_a^{bc} \cup \mathcal{H}_b^{ac} \cup \mathcal{H}_c^{ab}. \quad (21)$$

Likewise, given  $\mathcal{H}_{[abcd]} \equiv \mathcal{H}_a \otimes \mathcal{H}_b \otimes \mathcal{H}_c \otimes \mathcal{H}_d$  then

$$\begin{aligned} \mathcal{H}^{abcd} \equiv & \mathcal{H}_{[abcd]} - \mathcal{H}_{abcd} \cup \mathcal{H}_{ab}^{cd} \cup \mathcal{H}_{ac}^{bd} \cup \mathcal{H}_{ad}^{bc} \cup \mathcal{H}_{bc}^{ad} \cup \mathcal{H}_{bd}^{ac} \cup \mathcal{H}_{cd}^{ab} \cup \\ & \mathcal{H}_a^{bcd} \cup \mathcal{H}_b^{acd} \cup \mathcal{H}_c^{abd} \cup \mathcal{H}_d^{abc} \cup \mathcal{H}^{ab \bullet cd} \cup \mathcal{H}^{ac \bullet bd} \cup \mathcal{H}^{ad \bullet bc}. \end{aligned} \quad (22)$$

We will refer to sets such as  $\mathcal{H}^{abc}$  as a rank-3 entanglement, and so on. It is clear that in general, higher rank entanglements such as  $\mathcal{H}^{abcd}$  in the above require a deal of filtering out of separations from the original tensor product Hilbert space for their definition to be possible, and this accounts partly for the fact that entanglements are generally not as conceptually simple or intuitive as pure separations. It is not surprising that classical mechanics is easier to visualize than its quantum counterpart, because the former deals with separations exclusively whilst the latter deals with separations and entanglements.

We shall refer to the unique decomposition of a quantum register into the union of disjoint separations and entanglements as the *natural lattice*  $\mathfrak{L}(\mathcal{H})$  of  $\mathcal{H}$ , each element of which will be referred to as a *partition*. In general, partitions are separation products of entanglements and separations of various ranks, such that for each partition, the sum of the ranks of its factors equals the number of quantum subregisters. The individual separations and entanglements making up a partition will be called *blocks*.

The relationships between separations and entanglements are subtle and will be discussed in subsequent papers. The relationships between splits and partitions are equally important. Although the number of partitions in the

natural lattice of a rank- $n$  quantum register is the same as the number of splits, i.e., the  $n^{th}$  Bell number, as can be readily proved, in general, splits and partitions cannot coincide. Every factor in a split is a vector space whereas no block in a partition is a vector space. Both splits and partitions are essential ingredients in the construction of causal set structure in our paradigm.

In the following, we shall use the above index notation to label the various elements of entanglements and separations. For example,  $\psi_{123}^{456 \bullet 78}$  is interpreted to be some element in  $\mathcal{H}_{123}^{456 \bullet 78}$  and so on. With this notation we are entitled to rewrite the vector  $\psi_{123}^{456 \bullet 78}$  in the factorized form

$$\psi_{123}^{456 \bullet 78} = \psi_1 \otimes \psi_2 \otimes \psi_3 \otimes \psi^{456} \otimes \psi^{78}, \quad (23)$$

where  $\psi_1 \in \mathcal{H}_1$ ,  $\psi_2 \in \mathcal{H}_2$ ,  $\psi_3 \in \mathcal{H}_3$ ,  $\psi^{456} \in \mathcal{H}^{456}$  and  $\psi^{78} \in \mathcal{H}^{78}$ .

## V. PROBABILITY AMPLITUDES, FAMILY STRUCTURE AND CAUSAL SETS

We are now in a position to discuss causal sets proper. An important and natural feature of the stages paradigm (see Appendix) is that the state of the universe can change its factorizability as it jumps from  $\Psi_n$  to  $\Psi_{n+1}$  and this is the origin of family structure, as we shall demonstrate.

To every stage  $\Omega_n$  of the universe, we can assign a positive integer  $\mathcal{F}_n$ , the *current factorizability of the universe*. This is just the number of factor states in  $\Psi_n$  and we may write

$$\Psi_n = \Psi_n^1 \otimes \Psi_n^2 \otimes \dots \otimes \Psi_n^{\mathcal{F}_n}, \quad (24)$$

where the subscript  $n$  refers to exo-time (unphysical, i.e., unobservable, external time) and the superscripts now label the various factors, rather than denoting entanglements, there being  $\mathcal{F}_n$  of these factors. The various factor states  $\{\Psi_n^\alpha : 1 \leq \alpha \leq \mathcal{F}_n\}$  form a discrete set referred to as the *factor lattice*  $\Lambda_n$ . Although the dimension of the total Hilbert space is fixed, the number of elements in the factor lattice is time dependent and this forms the correct basis for a discussion of quantum causal sets.

In the conventional causal set paradigm, the discussion is in terms of a classical structure of sets related in various ways. We shall call such a model a *CCM* (classical causal model), whereas our paradigm will be referred to as a *QCM* (quantum causal model).

In a *CCM*, there is a concept of *internal temporality*, which means that each element is born either to the future of, or is unrelated to, all existing

elements; that is, no element can arise to the past of an existing element. In our *QCM*, every realized or potential stage is the outcome of some actual or potential test, and cannot be regarded as in the past of any realized or potential stage which is the origin of that test. Therefore the stages paradigm also has built-in internal temporality.

In *CCMs*, the *irreflexive convention* states that an element of a causal set does not precede itself. In our *QCM*, this is another expression of the consequences of the Kochen-Specker theorem [22], consistent with the notion that for quantum states, classical values such as position and momentum do not exist prior to measurement.

In causal set theory, a *link* is an irreducible relation, i.e., one not implied by other relations via transitivity. Such a relation is also referred to as a *covering relation* in the mathematical literature. In our *QCM*, links are directly related to outcomes of tests and the corresponding quantum amplitudes.

Consider the inner product between any two states in a quantum register. Because of the natural lattice structure of any quantum register, each state is in a specific partition and, depending on the details of the partitions concerned, the amplitude may or may not factorize. This is because factor states can only take inner products in combinations which lie in the same factor Hilbert space of some split of the total Hilbert space (quantum “zipping”). The following example illustrates the point:

**Example 1:** Consider the quantum register  $\mathcal{H}_{[1\dots 8]}$ . By inspection, the inner product of the states  $\psi_{123}^{456\bullet 78}$  and  $\phi_{145}^{23\bullet 678}$  takes the factorized form

$$\begin{aligned} \langle \phi_{145}^{23\bullet 678} | \psi_{123}^{456\bullet 78} \rangle &= \langle \phi_1 | \psi_1 \rangle \langle \phi^{23} | \psi_{23} \rangle \langle \phi_{45}^{678} | \psi^{456\bullet 78} \rangle \\ &= \langle \phi_1 | \psi_1 \rangle \langle \phi^{23} | \{ |\psi_2\rangle \otimes |\psi_3\rangle \} \\ &\quad \times \{ \langle \phi^{678} | \otimes \langle \phi_4 | \otimes \langle \phi_5 | \} \{ |\psi^{456}\rangle \otimes |\psi^{78}\rangle \} , \end{aligned} \quad (25)$$

which cannot be simplified further.

It is at this point that we find the natural and logical motivation for the concept of *family* in quantum causal set theory. The individual elements of a family are none other than groups of entanglements and separations associated with each separate factor in transition amplitudes.

Both splits and partitions are involved in such amplitudes, which can be more readily appreciated when we represent them graphically. The graphical notation we introduce here also turns out to be very useful in discussing causal set dynamics. The convention is to represent each possible factor in each state involved in an amplitude by a large circle, whilst common split



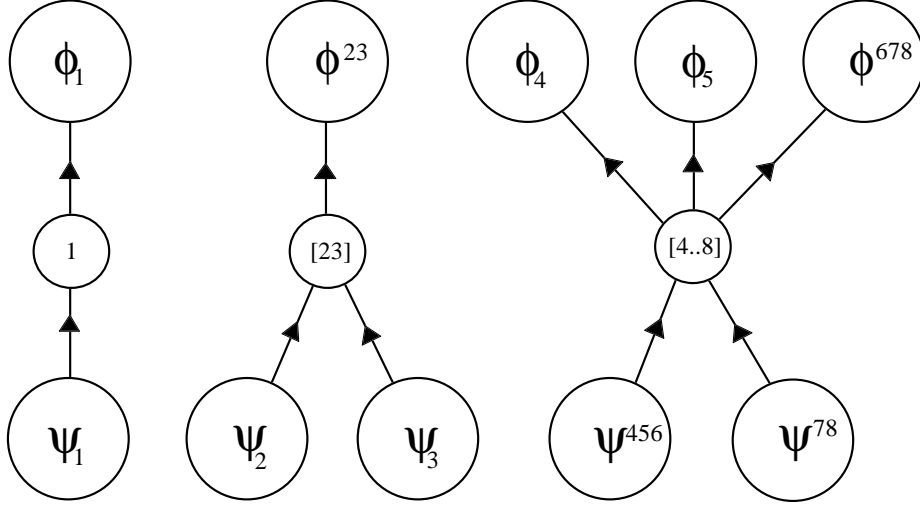


Figure 1: A typical quantum amplitude between states in a quantum register, showing the appearance of family structure.

factors linking different but compatible blocks are represented by smaller circles. Ket vectors represent earlier states and the diagram is drawn with time running upwards. With this convention, the amplitude in Example 1 is represented by Figure 1.

The next step is to incorporate the above to the stages paradigm. We shall take it for granted that the total Hilbert space  $\mathcal{H}_{[1\dots N]}$  is some vast tensor product of  $N$  elementary subregisters. Next, suppose that the current present state of the universe  $\Psi_n$  has  $k$  factors, i.e.,  $\Psi_n = \psi_1 \otimes \psi_2 \otimes \dots \otimes \psi_k$ ; each of which is a separation or entanglement. Without loss of generality, we may take each of these factors to have unit norm relative to the factor Hilbert space it lies in. Suppose now that the next test of the universe  $O_{n+1}$  is determined and that  $\Theta$  is some normalized eigenstate of the corresponding observable  $\hat{O}_{n+1}$ . Then  $\Theta$  is a possible candidate for  $\Psi_{n+1}$ , the next state of the universe after  $\Psi_n$ . According to the principles discussed in the Appendix, the probability of this outcome relative to  $\Psi_n$  is the conditional probability

$$P\left(\Psi_{n+1} = \Theta | \Psi_n, \hat{O}_{n+1}\right) = |\langle \Theta | \Psi_n \rangle|^2. \quad (26)$$

Suppose further that  $\Theta$  itself separates into  $l$  factors, i.e.  $\Theta = \theta_1 \otimes \theta_2 \otimes \dots \otimes \theta_l$ , each of which is normalized to unity. If the pattern of subregisters

associated with the corresponding partitions in which  $\Psi_n$  and  $\Theta$  lie is such that  $P(\Psi_{n+1} = \Theta | \Psi_n, \hat{O}_{n+1})$  itself factorizes into a number of factors, i.e.,

$$P(\Psi_{n+1} = \Theta | \Psi_n, \hat{O}_{n+1}) = P_1 P_2 \dots P_r, \quad r \leq \min(k, l), \quad (27)$$

then each of these factors  $P_i$  can be interpreted as a conditional transition probability within a distinct family.

**Example 2:** If in Example 1 we take  $\Psi_n \equiv \psi_{123}^{456 \bullet 78}$  and  $\Theta \equiv \phi_{145}^{23 \bullet 678}$ , then  $N = 8$ ,  $k = l = 5$ ,  $r = 3$  and

$$P_1 = |\langle \phi_1 | \psi_1 \rangle|^2, \quad P_2 = |\langle \phi^{23} | \psi_{23} \rangle|^2, \quad P_3 = |\langle \phi_{45}^{678} | \psi^{456 \bullet 78} \rangle|^2. \quad (28)$$

From examples such as this one, we are led to give the following definition of what we mean by family structure in the stages paradigm.

**Definition 1:** Given a quantum register consisting of two or more sub-registers, then in any quantum transition  $|\Psi_n\rangle \rightarrow |\Psi_{n+1}\rangle$ , the number of families involved in that transition is the number of factors in the transition amplitude  $\langle \Psi_{n+1} | \Psi_n \rangle$ , as determined via the sub-register structure of the register. Each of these factors identifies a unique family.

**Comment 1:** Each factor in a transition amplitude involves a distinct subset of the quantum subregisters making up the total Hilbert space. Therefore, the factors collectively in such a transition amplitude define a particular split of the total Hilbert space.

Once a family has been identified, it is possible to define *parents*, *offspring* and *siblings*:

**Definition 2:** In a given family transition, all the factors of the initial state of the family are *parents*, whilst the corresponding factors in the final state are *offspring*, and are *siblings* of each other if there are two or more such factors.

In example 2,  $\psi_1$  is the single parent of  $\phi_1$ , which has no siblings.  $\psi_2$  and  $\psi_3$  are the parents of the entangled state  $\phi^{23}$ , and  $\psi^{456}$  and  $\psi^{78}$  are the parents of  $\phi_4, \phi_5$  and  $\phi^{678}$ , which are siblings.

In general, extended sets of transitions, such as  $\Psi_n \rightarrow \Psi_{n+1} \rightarrow \Psi_{n+2} \rightarrow \dots$ , etc., will generate all the properties of causal sets, such as grandparents, grandchildren, and suchlike. Individual families may merge with other

families or persist with reasonably stable identities, depending on the specific transitions involved. As we discuss in a later section, the details of the potential transitions will be determined by the quantum tests involved.

Because we are dealing with quantum mechanics over a large rank quantum register, there will be in general many possible outcomes of each test. Each of these outcomes will then influence which future tests are chosen, so the range of potential futures grows enormously the more jumps we consider. The set of all alternative causal sets (potential futures) may be discussed in terms of a grand causal set structure, analogous to the concept of poset referred to in [5]. It is here that the concept of entropy will play a significant role. We will report on this elsewhere, noting that there will in general be two contributions to the entropy related to a single jump: the entropy associated with all the possible outcomes of any given test, and the entropy associated with all the possible tests which could occur with the given present.

Causal set structure emerges once we start to deal with more than two jumps. To illustrate the sort of causal set structures which quantum registers can generate, consider a Hilbert space  $\mathcal{H}_{[123456]}$  over 6 subregisters and the following sequence of normalized states:

$$\Psi^{123456} \rightarrow \psi_1^{23\bullet456} \rightarrow \theta_{16}^{24\bullet35} \rightarrow \eta_4^{12\bullet356} \rightarrow \phi^{12\bullet34\bullet56} \rightarrow \dots \quad (29)$$

From the factorization structure of the probabilities  $P(\psi_1^{23\bullet456}|\Psi^{123456}) \equiv |\langle\psi_1^{23\bullet456}|\Psi^{123456}\rangle|^2$ , etc., we can draw the Hasse diagram structure shown in Figure 2.

In this diagram we see several key features worth commenting on.

1. The initial state of the universe  $\Psi^{123456}$  is fully entangled. Because in our paradigm separability is a marker of classicality, the initial state has no classical attributes. It is not possible even to think in terms of observers and systems at this stage;
2. After the first jump, the universe has started to acquire classicality, in the form of separability of its state. We refer to this as the “quantum Big Bang”. In our paradigm, the “Big Crunch” is synonymous with a return to full entanglement;
3. In the second jump, the factor  $\psi_1$  changes to  $\theta_1$ , with no connection with any other part of the universe. To all intents and purposes, the universe appears to have split into two separate sub-universes having nothing

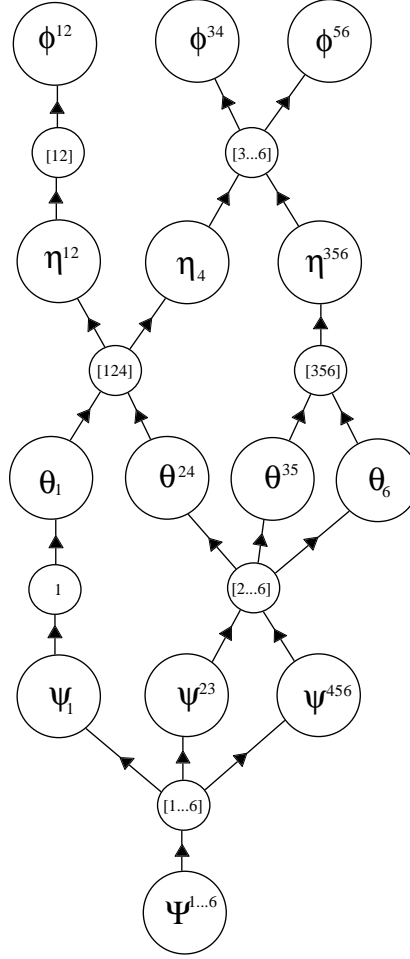


Figure 2: An example of causal set structure determined by successive states

to do with each other. In a universe with very many subregisters, it should be possible to use this sort of process to discuss quantum black hole physics and the issue of information loss into the interior of a black hole;

4. If it happened to be the case that  $\theta_1 = \psi_1$ , then to all intents and purposes that part of the universe would appear to have its local “end-time” frozen whilst the rest of the universe evolved. This constitutes what we call a local null test. Essentially, such a test leaves the state that it is testing unchanged, i.e., the initial state happens to be an eigenstate of the test. An example of this is the passage of an electron prepared via the spin-up channel of one Stern-Gerlach device through

another identically orientated Stern-Gerlach device.

This mechanism has a number of important consequences. One of these is that it permits a description of stage dynamics in terms of a local concept of time, “endo-time”, which unlike the external time used to label successive stages of the universe can have physical significance, in that it relates changes in factors of the state of the universe to each other. Endo-time is non-integrable, unlike exo-time, because the number of physically significant jumps any part of the universe undergoes depends on the details of those jumps. In our paradigm, local null tests are expected to be the origin of proper time in relativity (which is regarded here as an emergent theory);

5. Another consequence of null tests is that despite the absolute nature of exo-time (which labels states of the universe), there is no reason to suppose that endo-time is absolute. For instance, in Figure 2, if  $\psi_1 = \theta_1$ , we could regard  $\psi_1$  as simultaneous with  $\theta^{24}$ ,  $\theta^{35}$  and  $\theta_6$  without any contradictions arising. Likewise, we could regard  $\theta_1$  as simultaneous with  $\psi^{23}$  and  $\psi^{456}$ .

In the long run it should be possible to use local null tests and very large rank quantum registers to explain the emergence of relativistic physics and the equivalence of local inertial frames. This would require in addition a careful account of the process physics of quantum measurement, none of which is generally given in conventional theory. For instance, whilst it may be meaningful to give an *a posteriori* relativistically covariant discussion of *expectation values*, the same cannot be said of any individual outcome of a single run of a quantum experiment, because a single quantum outcome cannot be “observed” by two different observers using different equipment.

6. A form of ‘lightcone’ structure can be seen to occur when we look at the relationship between  $\theta_1$  and  $\eta^{356}$ . These are incomparable elements in the language of causal sets. The latter factor looks as if it would be unchanged by any counterfactual change in the former. However, we caution against taking this aspect of the Hasse diagram too literally here. The diagram we are dealing with in Figure 2 involves factors in amplitudes, which are known to have non-local correlations. What is not shown in Figure 2 is another causal set, the one associated with the tests which give the outcomes discussed here. If indeed we considered a change in  $\theta_1$ , it is quite possible that subsequent tests did not even have any factorizable outcomes, i.e.  $\eta^{356}$  need not exist as part of a potential future of an altered  $\theta_1$ .

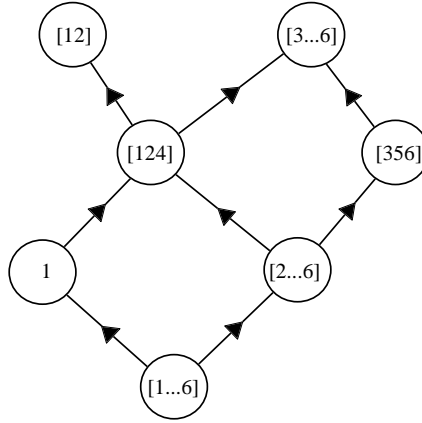


Figure 3: Reduced causal set structure involving successive splits in Figure 2.

7. Family structure per se is determined by the structure of successive splits of the quantum register. The causal set structure related to this aspect of the dynamics can be identified if the circles representing state factors are replaced by lines, leaving the smaller circles representing split factors. This shows more clearly how different families relate to each other, rather than their individual family members. For example, Figure 2 gives the reduced diagram Figure 3.

## VI. EINSTEIN LOCALITY AND CAUSAL SETS

The appearance of family structures relating factors in successive states of the universe, together with the very large number of factors observed in the current epoch of the universe, opens the door to a discussion of metrical and other classical concepts. Such a discussion is needed because emergent space-time appears to have all the hallmarks of a classical pseudo-Riemannian manifold, with an integral dimension, a Lorentzian signature metric and curvature induced by the local presence of matter. A particular feature of separability is distinguishability, that is, the various factors of a separable state can be identified directly with the corresponding quantum subregisters concerned. These registers are regarded as distinct, and therefore, this may be regarded as the origin of classicality (i.e., the ability to distinguish objects).

The same cannot be said of entangled states. In classical mechanics, for instance, it is well known that states of systems cannot be entangled, because their properties have to be well defined.

We envisage that many intricate patterns and hierarchies of patterns of related factors could persist in approximate detail as the universe jumps from stage to stage, thereby creating the appearance of a universe with semi-classical structures, analogous to various patterns seen in Conway’s “Game of Life”, for example. These structures could have approximate relationships describable in terms of space, distance and other classical constructs by endo-physical observers, themselves described by such patterns of factors. Underlying this description would be the counting procedures used by such endo-physical observers to register approximate estimates of jumps (giving rise to measurements of emergent time) and approximate estimates of family relationships (giving rise to emergent concepts of space). Exactly how emergent spacetime and matter could arise from causal set familial relationships and persistence is an enormous problem reserved for the future.

The observation of quantum correlations creates a problem for such an emergent picture, however. Although much of physics appears to respect Einstein locality [14] as far as causality is concerned, the non-local superluminal quantum correlations observed in *EPR* type experiments appear incompatible with the principles of relativity. This is consistent with the notion that there are really two different causal mechanisms involved, each with its own variety of “distance” relationships. One mechanism is involved with Einstein locality and all that it implies, such as a maximum speed (the speed of light) for the propagation of physical information such as energy and momentum, whereas the other mechanism is responsible for the transmission of quantum correlations, which appear to occur with no limitation of speed in any frame [23]. The problem for conventional physics based on Lorentzian manifolds is that it cannot “explain” the latter mechanism. If Einstein locality is synonymous with classical Lorentzian manifold structure, and if this structure is emergent, then it seems reasonable to interpret quantum correlations as a signal that there is a pre-geometric (or pre-emergent) structure underlying the conventional spacetime paradigm.

In the stages paradigm, there naturally occurs two different components involved in the dynamical evolution. These are the tests and the outcomes of those tests respectively and the properties and characteristics of each of these components differ. Our hypothesis is that the tests are responsible for the appearance of Einstein locality, whereas it is the outcomes which display non-local effects such as quantum correlations.

Conventional quantum field theory is consistent with this point of view. There too there are two aspects to the dynamics, i.e., the quantum field

operators out of which the dynamical observables are constructed and the quantum states. For various technical reasons, the Heisenberg picture is generally the one employed in quantum field theory. In this picture, states are frozen in time between state preparation and measurement, whilst dynamical evolution is locked into the field operators. In this picture, which happens also to be the best picture to discuss the stages paradigm, quantum field operators satisfy classical equations of motion for their evolution over a classical spacetime, i.e., they obey operator Heisenberg equations of motion. This does not imply that super-luminal quantum correlations cannot take place, because such correlations relate to the properties of quantum states and not to the observables (the operators of physical interest) of the theory. Local observable densities such as energy and momentum density operators satisfy microscopic causality [24], which means that they have commutators which vanish at relative spacelike intervals, even if the local fields out of which they are constructed do not. This guarantees that Einstein locality holds as far as the corresponding classical variables are concerned.

The relationship between quantum field theory and the stages paradigm is even stronger. In scattering quantum field theory for example, there is the same structure of state preparation, test and outcome as in the stages paradigm. First an “in” state  $|\Psi\rangle_{in}$  is prepared at what amounts to the remote past. In that regime, the “in” state is taken to be a many-particle eigenstate of some beam preparation apparatus. This apparatus will be associated with some preferred basis set  $\mathcal{B}_{in}$ , one element of which is selected to be  $|\Psi\rangle_{in}$ . The system is then left alone until at what amounts to infinite future time, it is tested against what is equivalent to a preferred basis  $\mathcal{B}_{out}$  of free particle states, the “out” states. Because an understanding of the Hilbert space of possible states in fully interacting quantum field theory is virtually non-existent, the Heisenberg picture is invariably the one used in scattering theory. The traditional vehicle for calculation is the S-matrix formalism [24, 25], which gives the transition probabilities generated by some unitary transformation of the final state basis  $\mathcal{B}_{out}$  relative to the initial (preparation) state basis  $\mathcal{B}_{in}$ . In the Heisenberg picture, it is not the case that the state being tested changes unitarily in time. Rather, time is something associated with the tests constructed by the observer.

## VII. SKELETON SETS

In the stages paradigm, the tests involved in jumps are not arbitrary but are represented by specific operators determined by as yet unknown dynamical principles. In particular, these operators are assumed to be Hermitian,



because the principles of standard  $QM$  are based on such operators. In general, if we are dealing with a finite dimensional Hilbert space of dimension (say)  $n$ , then we can find  $n^2$  independent operators [14] out of which we can build all possible Hermitian operators. This leads us to a discussion of skeleton sets of operators.

To explain the notion of a skeleton set of operators, consider the simple example of a qubit register, i.e., a two-dimensional Hilbert space  $\mathcal{H}_A$ , where the subscript  $A$  labels the qubit, in anticipation of a generalization to a many-qubit register. Given a preferred basis  $\mathcal{B}_A \equiv \{|1\rangle_A, |2\rangle_A\}$  for  $\mathcal{H}_A$  there are four Hermitian operators  $\hat{\sigma}_A^\mu : \mu = 0, 1, 2, 3$  which may be used to construct any Hermitian operator on  $\mathcal{H}_A$ . Here  $\hat{\sigma}_A^0$  is the identity operator on  $\mathcal{H}_A$  and the  $\hat{\sigma}_A^i : i = 1, 2, 3$  are three operators analogous to the Pauli matrices, satisfying the product rules

$$\begin{aligned}\hat{\sigma}_A^i \hat{\sigma}_A^j &= \delta_{ij} \hat{\sigma}_A^0 + i \varepsilon_{ijk} \hat{\sigma}_A^k, \\ \hat{\sigma}_A^i \hat{\sigma}_A^0 &= \hat{\sigma}_A^0 \hat{\sigma}_A^i = \hat{\sigma}_A^i,\end{aligned}\tag{30}$$

where we use the summation convention with small Greek and small Latin symbols, but not on the large Latin indices which label qubits. These rules may be written in the more compact form

$$\hat{\sigma}_A^\mu \hat{\sigma}_A^\nu = c^{\mu\nu}_\alpha \hat{\sigma}_A^\alpha,\tag{31}$$

where the  $c^{\mu\nu}_\alpha$  are given by

$$\begin{aligned}c^{0\nu}_\mu &= c^{\nu 0}_\mu \equiv \delta_{\nu\mu}, \\ c^{ij}_0 &\equiv \delta_{ij} \\ c^{ij}_k &\equiv i \varepsilon_{ijk}.\end{aligned}\tag{32}$$

In the standard basis  $\mathfrak{B}_A$  we may represent these operators by

$$\hat{\sigma}_A^\mu \equiv \sum_{i,j=1}^2 |i\rangle_A [\sigma_A^\mu]_{ij} \langle j|,\tag{33}$$

where the matrices  $[\sigma_A^\mu]$  are defined by

$$\begin{aligned}[\sigma_A^0] &\equiv \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, & [\sigma_A^1] &\equiv \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \\ [\sigma_A^2] &\equiv \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}, & [\sigma_A^3] &\equiv \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}.\end{aligned}\tag{34}$$

The four operators  $\hat{\sigma}_A^\mu$  will be called a *skeleton set* for  $\mathcal{H}_A$  and denoted by  $\mathcal{S}_A$ . The significance of this set arises from the fact that an arbitrary linear combination of elements of  $\mathcal{S}_A$  with real coefficients is a Hermitian operator. Moreover, any Hermitian operator on  $\mathcal{H}_A$  can be uniquely represented as a real linear combination of elements of the skeleton set. The skeleton set  $\mathcal{S}_A$  therefore forms a basis for the real vector space  $\mathbb{H}(\mathcal{H}_A)$  of (linear) Hermitian operators on  $\mathcal{H}_A$ . In addition, this vector space is closed under multiplication based on the rule (33) and therefore forms an (linear) algebra.

Going further, consider a quantum register  $\mathcal{H}$  consisting of  $N$  qubits, given by the tensor product

$$\mathcal{H} \equiv \mathcal{H}_1 \otimes \mathcal{H}_2 \otimes \dots \otimes \mathcal{H}_N. \quad (35)$$

We define a skeleton set  $\mathcal{S}$  for  $\mathcal{H}$  by the direct product

$$\mathcal{S} \equiv \{\hat{\sigma}_1^{\mu_1} \otimes \hat{\sigma}_2^{\mu_2} \otimes \dots \otimes \hat{\sigma}_N^{\mu_N} : 0 \leq \mu_1, \mu_2, \dots, \mu_N \leq 3\}. \quad (36)$$

These are all Hermitian operators on  $\mathcal{H}$  and form a basis for the real vector space  $\mathbb{H}(\mathcal{H})$  of Hermitian operators on  $\mathcal{H}$ . An arbitrary element  $\hat{O}$  of  $\mathbb{H}(\mathcal{H})$  is of the form

$$\hat{O} \equiv O_{\mu_1 \mu_2 \dots \mu_N} \hat{\sigma}_1^{\mu_1} \otimes \hat{\sigma}_2^{\mu_2} \otimes \dots \otimes \hat{\sigma}_N^{\mu_N}, \quad (37)$$

where we sum over the small Greek indices and the coefficients  $O_{\mu_1 \mu_2 \dots \mu_N}$  are all real. Moreover, multiplication of elements of  $\mathbb{H}(\mathcal{H})$  is defined in a straightforward way. We have

$$\begin{aligned} \hat{A}\hat{B} &\equiv \{A_{\mu_1 \mu_2 \dots \mu_N} \hat{\sigma}_1^{\mu_1} \otimes \hat{\sigma}_2^{\mu_2} \otimes \dots \otimes \hat{\sigma}_N^{\mu_N}\} \{B_{\nu_1 \nu_2 \dots \nu_N} \hat{\sigma}_1^{\nu_1} \otimes \hat{\sigma}_2^{\nu_2} \otimes \dots \otimes \hat{\sigma}_N^{\nu_N}\} \\ &= A_{\mu_1 \mu_2 \dots \mu_N} B_{\nu_1 \nu_2 \dots \nu_N} c_{\alpha_1}^{\mu_1 \nu_1} c_{\alpha_2}^{\mu_2 \nu_2} \dots c_{\alpha_N}^{\mu_N \nu_N} \hat{\sigma}_1^{\alpha_1} \otimes \hat{\sigma}_2^{\alpha_2} \otimes \dots \otimes \hat{\sigma}_N^{\alpha_N}, \end{aligned} \quad (38)$$

which is some element of  $\mathbb{H}(\mathcal{H})$ , which means that  $\mathbb{H}(\mathcal{H})$  is an algebra [12] over the real number field.

Given a large  $N$  qubit register  $\mathcal{H}$ , then in the stages paradigm, dynamical evolution on the pre-geometric level involves a succession of tests  $O_n$  and associated outcomes  $\Psi_n$ . In this paradigm, the total Hilbert space  $\mathcal{H}$  is fixed, in contrast to some other models [10, 26]. Assuming  $N$  is extremely large and finite, then there will be a natural skeleton set  $\mathcal{S}$  given in (36) which permits a decomposition of each test, i.e., we may write

$$\hat{O}_n = C_{\mu_1 \mu_2 \dots \mu_N}^n \hat{\sigma}_1^{\mu_1} \otimes \hat{\sigma}_2^{\mu_2} \otimes \dots \otimes \hat{\sigma}_N^{\mu_N}, \quad (39)$$

where the coefficients  $C_{\mu_1 \mu_2 \dots \mu_N}^n$  are all real. Here and elsewhere we shall use the summation convention.

We have already discussed the analogy between the behaviour of tests in the stages paradigm and operators in quantum field theory. It may be the

case that for a given  $n$ , the set of coefficients  $\{C_{\mu_1\mu_2\ldots\mu_N}^n\}$  is such that the right hand side in (39) factorizes, i.e., we may write

$$\hat{O}_n = \hat{A} \otimes \hat{B}, \quad (40)$$

where  $\hat{A}$  is an operator acting on one factor subspace  $\mathcal{H}_A$  of  $\mathcal{H}$  and  $\hat{B}$  acts on the other factor subspace  $\mathcal{H}_B$ . Together,  $\mathcal{H}_A$  and  $\mathcal{H}_B$  give a bi-partite factorization or split of  $\mathcal{H}$  [21], i.e.

$$\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B. \quad (41)$$

We now discuss the necessary and sufficient conditions for a Hermitian operator on  $\mathcal{H}$  to factorize with respect to the basic skeleton set.

**Theorem 1:** Let  $\mathcal{H} \equiv \mathcal{H}_1 \otimes \mathcal{H}_2$  be an 2-qubit quantum register with standard basis

$$\mathcal{B} \equiv \{|i_1\rangle_1 \otimes |i_2\rangle_2 : 1 \leq i_1, i_2 \leq 2\} \quad (42)$$

and standard skeleton set  $\mathcal{S} \equiv \{\hat{\sigma}_1^{\mu_1} \otimes \hat{\sigma}_2^{\mu_2} : 0 \leq \mu_1, \mu_2 \leq 3\}$  and let  $\mathbb{H}(\mathcal{H})$  be the set of Hermitian operators on  $\mathcal{H}$ . Then an arbitrary element  $\hat{O} \in \mathbb{H}(\mathcal{H})$ , given by an expansion of the form

$$\hat{O} = c_{\mu\nu} \hat{\sigma}_1^\mu \otimes \hat{\sigma}_2^\nu, \quad c_{\mu\nu} \in \mathbb{R}, \quad (43)$$

factorizes with respect to  $\mathcal{S}$  if and only if the coefficients  $c_{\mu\nu}$  satisfy the *micro-singularity* condition

$$c_{\mu\nu} c_{\alpha\beta} = c_{\mu\beta} c_{\alpha\nu} \quad (44)$$

for all values of the indices.

**Proof:**  $\Rightarrow$ : If an operator  $\hat{O} \in \mathbb{H}(\mathcal{H})$  factorizes relative to  $\mathcal{S}$  then we may write

$$\begin{aligned} \hat{O} &= (a_\mu \hat{\sigma}_1^\mu) \otimes (b_\nu \hat{\sigma}_2^\nu) \\ &= (a_\mu b_\nu) \hat{\sigma}_1^\mu \otimes \hat{\sigma}_2^\nu, \end{aligned} \quad (45)$$

which means we take  $c_{\mu\nu} = a_\mu b_\nu$  for all values of the indices, and these clearly satisfy the micro-singularity condition (44).

$\Leftarrow$ : Suppose  $\hat{O} \in \mathbb{H}(\mathcal{H})$  such that the coefficients of its expansion (43) satisfy the micro-singularity condition (44). Without loss of generality we may assume  $\hat{O}$  is not the zero operator. Therefore, there is at least

one coefficient  $c_{\alpha\beta}$  in the expansion (43) which is non-zero. Hence we may write

$$\begin{aligned} c_{\alpha\beta}\hat{O} &= c_{\alpha\beta}c_{\mu\nu}\hat{\sigma}_1^\mu \otimes \hat{\sigma}_2^\nu \\ &= c_{\alpha\nu}c_{\mu\beta}\hat{\sigma}_1^\mu \otimes \hat{\sigma}_2^\nu, \quad \text{using (44)} \\ &= (c_{\mu\beta}\hat{\sigma}_1^\mu) \otimes (c_{\alpha\nu}\hat{\sigma}_2^\nu), \end{aligned} \tag{46}$$

which proves  $\hat{O}$  is separable with respect to  $\mathcal{S}$ . This result generalizes to more general sub-registers than just qubits.

There are fundamental differences between the concept of entanglement of states (for which a similar micro-singularity theorem holds [21]) and the “entanglement” of Hermitian operators. First, the former involves the complex numbers whereas the latter involves the reals. Quantum probabilities are extracted by taking the square moduli of inner products of states, a process which generates the phenomenon of quantum interference, an inherently quantum effect. No such phenomenon arises with Hermitian operators, where at best only products of operators are ever constructed (the operators form a ring). In general, there seems to be no physically motivated concept of inner product on the space  $\mathbb{H}(\mathcal{H})$  of Hermitian operators over a Hilbert space, and no corresponding probability interpretation, although the stages paradigm emphasizes the possibility that perhaps there should be such a thing.

The difference between states and observables should manifest itself in the sort of causal sets they are associated with. In quantum mechanics generally, states satisfy the principle of superposition, which leads to non-local and non-classical consequences at odds with classical relativity, whereas observables tend to have classical analogues which satisfy Einstein locality and obey classical equations of motion consistent with relativity. We recall that canonical quantization is the standard procedure of replacing classical variables with their quantum operator counterparts. In standard quantum mechanics, those operators usually satisfy the same causal relations as their classical analogues. For example, those operators identified as observables should commute at spacelike separations.

In the stages paradigm, we do not yet know the rules the universe uses for choosing tests. However, because those tests should correspond to standard observables in the appropriate emergent limits, one way of ensuring this is if the causal set structure associated with successive tests follows patterns analogous to classical cellular automata. A particular feature of automata based on nearest neighbour interactions is the appearance of zones of causal influence looking very much like lightcones in relativity and fully consistent with Einstein causality.

In support of this line of argument, we refer to Theorem 4, proved in the next section, which says that separability of a test necessarily implies separability of outcome. Because separability is a necessary attribute of classical space, Theorem 4 implies that separability of the observables begins to drive classicality in the states, as discussed in Example 3.

Another difference between tests and states is that in general, for a fixed dimension  $N$  of the quantum register  $\mathcal{H}$ , the dimensionality of the vector space  $\mathbb{H}(\mathcal{H})$  of Hermitian operators is  $N^2$ , which means that the operators have a richer structure in terms of their separability and entanglement than does the corresponding set of states.

Despite their expected differences, however, the relationship between the separability and entanglement properties of states and of operators is a deep and important one which is discussed next.

## VIII. EIGENVALUES

The question of eigenvalues of operators is a fundamental one, because the spectrum of a Hermitian operator is directly associated with physical information. This is also related to the concepts of separability and entanglement of operators and to the important issue of *preferred bases*.

In the following we shall assume all Hilbert spaces are finite dimensional and make references to the following terms:

A *degenerate* operator is a Hermitian operator with at least two linearly independent eigenstates with identical eigenvalues.

A *weak* operator is a Hermitian operator which is either degenerate or at least one of its eigenvalues is zero.

A *strong* operator is a Hermitian operator which is not weak; that is, none of its eigenvalues are zero and all its eigenvalues are distinct.

**Comment 2:** Degeneracy of eigenvalues represents a certain loss of information, in that different eigenstates with the same eigenvalue cannot be physically separated by the apparatus concerned. In this sense, eigenvalues are not important in absolute terms *per se*. What is important is the knowledge that one eigenstate is distinguishable from another, and that is why physicists generally try to construct tests represented by non-degenerate operators.

**Theorem 2:** All the normalized eigenstates of a strong operator  $\hat{O}$  form a unique, orthonormal basis set known as the *preferred basis* relative to  $\hat{O}$ , denoted by  $\mathfrak{B}(\hat{O})$ . The number of these eigenstates equals the dimension of the Hilbert space on which  $\hat{O}$  acts.

**Proof:** This is an extension of standard theory, the only difference being that we deal with strong operators rather than non-degenerate operators.

**Comment 3:** Projection operators are not strong.

**Comment 4:** Strong operators are important in the context of tensor product registers because weak operators are in themselves insufficient to determine a preferred basis, except in the particular case of a single qubit system, which is of no interest here.

Suppose now we have a tensor product Hilbert space  $\mathcal{H}_{[12]} \equiv \mathcal{H}_1 \otimes \mathcal{H}_2$  and suppose  $\hat{O}_1 \in \mathbb{H}(\mathcal{H}_1)$  and  $\hat{O}_2 \in \mathbb{H}(\mathcal{H}_2)$ . Then the tensor product operator  $\hat{O}_{12} \equiv \hat{O}_1 \otimes \hat{O}_2$  is a separable element of  $\mathbb{H}(\mathcal{H}_{[12]})$ . We observe the following:

- i) If either  $\hat{O}_1$  or  $\hat{O}_2$  is weak (as far as their eigenvalues with respect to their respective Hilbert spaces are concerned), then  $\hat{O}$  is necessarily degenerate.
- ii) If  $\hat{O}_1$  and  $\hat{O}_2$  are both strong, then  $\hat{O}_{12} \equiv \hat{O}_1 \otimes \hat{O}_2$  could be either strong or weak.

**Comment 5:** Every element of the skeleton set (36) associated with an  $n$ -qubit register is weak for  $n > 1$ .

**Definition 3:** Let  $\mathcal{S} \equiv \{a, b, \dots, z\}$  be a finite set of real numbers. Then  $\mathcal{S}$  is *strong* if the elements are distinct and none is zero.

**Definition 4:** The *pair-wise product*  $\mathcal{ST}$  of two finite real sets  $\mathcal{S} \equiv \{a, b, \dots, z\}$ ,  $\mathcal{T} \equiv \{A, B, \dots, Z\}$  is the set of all products of the elements of  $\mathcal{S}$  with the elements of  $\mathcal{T}$ , i.e.,

$$\mathcal{ST} \equiv \{aA, aB, \dots, zZ\}. \quad (47)$$

**Theorem 3:** A direct product  $\hat{O}_{12} \equiv \hat{O}_1 \otimes \hat{O}_2$  of two strong operators is strong if and only if the pair-wise product of their corresponding spectra is strong.

Conversely, if the tensor product of two Hermitian operators is strong, then each of these operators must also be strong.

**Proof:** This follows from the fact that an operator is strong if and only if its spectrum is strong, and from the observation that the spectrum of a tensor product operator is the pair-wise product of their respective spectra.

This leads to the following theorem which has important implications for the physics of entanglement:

**Theorem 4:** (The fundamental theorem) All the eigenstates of a separable strong operator operators are separable. Conversely, entangled states can be produced only by entangled operators.

**Proof:** Let  $\mathcal{H}_1$  and  $\mathcal{H}_2$  be two Hilbert spaces of dimension  $d_1, d_2$  respectively and let  $\hat{O}_1 \in \mathbb{H}(\mathcal{H}_1)$  and  $\hat{O}_2 \in \mathbb{H}(\mathcal{H}_2)$ . Then  $\hat{O}_{12} \equiv \hat{O}_1 \otimes \hat{O}_2 \in \mathbb{H}(\mathcal{H}_{[12]})$  is a separable operator. Given that  $\hat{O}$  is strong, then the following must be true:

- i) By Theorem 2,  $\hat{O}_{12}$  has precisely  $d_1 d_2$  distinct eigenstates;
- ii) By Theorem 3,  $\hat{O}_1$  and  $\hat{O}_2$  are each necessarily strong.

From ii), we can find a unique orthonormal basis  $\mathfrak{B}_1 \equiv \{u_1, \dots, u_{d_1}\}$  for  $\mathcal{H}_1$  consisting of the distinct eigenstates of  $\hat{O}_1$ . Likewise, we can find a unique orthonormal basis  $\mathfrak{B}_2 \equiv \{v_1, \dots, v_{d_2}\}$  for  $\mathcal{H}_2$  consisting of the distinct eigenstates of  $\hat{O}_2$ . Now consider the product states  $u_i \otimes v_j$ ,  $1 \leq i \leq d_1, 1 \leq j \leq d_2$ . There are exactly  $d_1 d_2$  such products, they are all separable, and each of these is an eigenstate of  $\hat{O}_{12}$ , as can be readily proved. But according to i),  $\hat{O}_{12}$  has only  $d_1 d_2$  distinct eigenstates. Therefore, all the eigenstates of  $\hat{O}_{12}$  are separable.

**Comment 6:** This result immediately generalizes to tensor products of  $n \geq 2$  operators. It leads to the fundamental conclusion that entangled states can be the outcomes of entangled operators only, a fact which has significant implications in physics. Physics laboratories which are not in any way correlated cannot create states entangled relative to those laboratories.

We may extend our graphical notation to include observables. Each completely entangled observable will be denoted by a square. A separable operator with  $k$  factors is represented by  $k$  squares aligned left to right. Lines with arrow going upwards and into such squares represent incoming prepared factor states whilst outgoing lines running upwards represent outcomes.

With this convention, then Theorem 4 states that diagrams such as Figure 4(a) are forbidden whereas processes represented by Figure 4(b) are permitted.

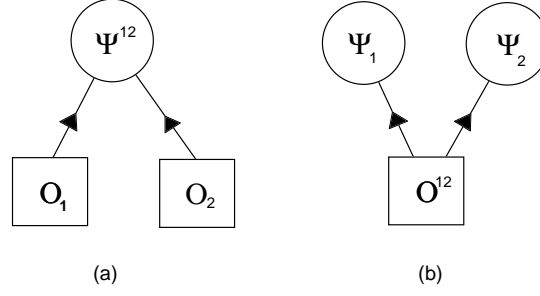


Figure 4: By Theorem 4, diagrams such as (a) are forbidden whereas those such as (b) are not.

### Time reversal

The stages paradigm has a built in irreversibility of time, because the process of state preparation, test and outcome cannot in general be reversed without altering the test. Suppose  $\Psi$  is a prepared state, being an outcome of test  $\hat{A}$ . Now consider a subsequent test  $\hat{B}$  of  $\Psi$ . If  $\Theta$  is an eigenstate of  $\hat{B}$  then the conditional probability  $P(\Theta|\Psi, \hat{B})$  of this outcome is given by the standard Born rule

$$P(\Theta|\Psi, \hat{B}) = |\langle\Theta|\Psi\rangle|^2. \quad (48)$$

The right hand side is invariant to the interchange of  $\Psi$  and  $\Theta$ , but the left hand side is not invariant to this change in general, because  $\Psi$  need not be an eigenstate of  $\hat{B}$ .

If we represent this sequence of events diagrammatically, as in Figure 5a, it is clear that merely interchanging the direction of the arrows need not be physically meaningful. Instead, the sequence of tests has to be reversed as well, as in Figure 5b, and then this gives the correct time-reversal equality

$$P(\Psi|\Theta, \hat{A}) = P(\Theta|\Psi, \hat{B}), \quad (49)$$

provided test  $\hat{A}$  is a legitimate test of state  $\Theta$ , which depends on the dynamics.



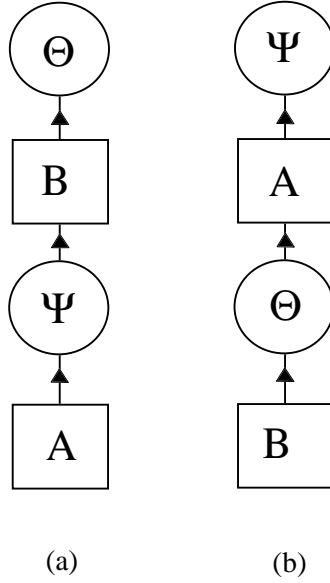


Figure 5: (b) is the time reversal of (a)

## IX. PHYSICAL EXAMPLES

An important fact that has physical consequences is that entangled operators can have entangled eigenstates **and** separable eigenstates, but it is not true the other way around, according to Theorem 4. Factorizable strong operators cannot have entangled eigenstates. Moreover, it is possible for an entangled operator to have only separable eigenstates.

Given that there exists a structure of separations and entanglements for observables (elements of  $\mathbb{H}(\mathcal{H})$ ) relative to skeleton sets of operators, then it is meaningful to talk about separable and entangled observables *per se*. Physically these concepts make sense. It is possible to construct an experiment which prepares a factorizable state, with each factor being produced by a separate piece of the apparatus. The apparatus can then be regarded as two identifiably distinct pieces of equipment, and therefore represented by a factorizable element of  $\mathbb{H}(\mathcal{H})$ . Likewise, it is possible to perform a single experiment consisting of two pieces of equipment placed at large spacelike separations, such that each part of the experiment acts on some aspect of an entangled state. The combined pieces of equipment can in some circumstances be regarded as separable, and in other circumstances, as an entangled pair. Certainly, if the equipment is designed to have entangled outcomes, as recent teleportation experiments are designed to do, then the two pieces of

equipment cannot be regarded as separable, because by our discussion above, such equipment could only produce separable states.

Theorem 4 underlines the fact that the operator causal set structure constrains the state causal set structure, as the following examples show.

**Example 3:** (Quantum Big Bang) Consider a universe consisting of a vast number  $N = 2^M$  of qubits, where  $M$  is an integer and assume that the stages paradigm holds. Then each state of the universe  $\Psi_n$  is an eigenstate of some test  $\hat{O}_n$  and the total quantum register  $\mathcal{H}_{[1\dots N]}$  has dimension  $2^{2^M}$ .

Suppose the state of the universe  $\Psi_n$  is fully entangled for  $n < 0$ . Then according to Theorem 4, it is necessarily the case that for  $n < 0$ ,  $\hat{O}_n$  is fully entangled, relative to the skeleton set associated with the register.

Now suppose that for each time  $n$  after zero, the stages dynamics is such that  $\hat{O}_n$  doubles the number of its factors, according to the scheme

$$\begin{aligned}\hat{O}_0 &= \hat{A}^{1\dots 2^M} \in \mathbb{H}(\mathcal{H}^{1\dots 2^M}), \\ \hat{O}_1 &= \hat{A}^{1\dots 2^{M-1}} \otimes \hat{A}^{(2^{M-1}+1)\dots 2^M} \in \mathbb{H}(\mathcal{H}^{1\dots 2^{M-1} \bullet (2^{M-1}+1)\dots 2^M}), \\ \hat{O}_2 &= \hat{A}^{1\dots 2^{M-2}} \otimes \hat{A}^{(2^{M-2}+1)\dots 2^{M-1}} \otimes \hat{A}^{(2^{M-1}+1)\dots 3 \times 2^{M-2}} \otimes \hat{A}^{(3 \times 2^{M-2}+1)\dots 2^M} \\ &\vdots \\ \hat{O}_M &= \hat{A}_1 \otimes \hat{A}_2 \otimes \dots \otimes \hat{A}_{2^M}, \quad \hat{A}_i \in \mathbb{H}(\mathcal{H}_i),\end{aligned}\tag{50}$$

up to the maximum possible. Then according to Theorem 4, whatever the outcome  $\Psi_n$  is, it must increase its separability at each stage after exo-time zero until it attains total factorizability at exo-time  $M$ . Unless  $\hat{O}_n$  entangles in any way after time  $M$ , the fate of this model universe is a “heat death”, where the universe ends up consisting of  $2^M$  non-interacting, totally isolated qubit sub-universes.

In this scheme, the operators are driven relentlessly to factorize after time zero, regardless of state outcome. Therefore there is no “feedback” from the outcomes. An alternative scheme with such a feedback would be to have factorization of the operators dependent on whether an outcome was itself separable or entangled.

**Example 4:** (EPR experiments) Suppose an entangled state  $\Psi \in \mathcal{H}^{12}$  is prepared via some observable  $\hat{O}$ . Then  $\hat{O}$  necessarily has to be entangled, according to Theorem 4.

Suppose now that subsequently,  $\hat{O}$  factorizes into two operators, i.e.  $\hat{O} \rightarrow \hat{A} \otimes \hat{B}$ . Then the outcome of this new test is necessarily separable,

i.e.  $\Psi \rightarrow \Theta_A \otimes \Theta_B$ , where  $\Theta_A$  is an eigenstate of  $\hat{A}$  (Alice) and  $\Theta_B$  is an eigenstate of  $\hat{B}$  (Bob). From this we see that quantum entanglement can be “unravelling” by ensuring that entangled states are tested by apparatus consisting of distinct pieces. Conventionally, this can be arranged by separating such pieces in physical space, but that is only something that can be considered in the emergent limit where physical space is a good approximation.

**Example 5:** (Superluminal correlations) Suppose we have a quantum register consisting of a large number  $2N$  of qubits, and suppose that at time  $n$ , the state  $\Psi_n$  of the universe consists of two factors,  $\psi^{1\dots N} \in \mathcal{H}^{1\dots N}$  and  $\phi^{(N+1)\dots 2N} \in \mathcal{H}^{(N+1)\dots 2N}$ . By our notation, this means that each of these is completely entangled with respect to its half of the quantum register. Suppose further that  $\psi^{1\dots N}$  is an eigenstate of factor operator  $\hat{A}^{1\dots N} \in \mathbb{H}(\mathcal{H}^{1\dots N})$  and that  $\phi^{(N+1)\dots 2N}$  is an eigenstate of factor operator  $\hat{B}^{(N+1)\dots 2N} \in \mathbb{H}(\mathcal{H}^{(N+1)\dots 2N})$ .

Now suppose that qubit  $N + 1$  joins qubits 1 to  $N$  to form a factor  $\hat{C}^{1\dots (N+1)} \in \mathbb{H}(\mathcal{H}^{1\dots (N+1)})$  in the next test  $\hat{O}_{n+1}$  of the universe and that  $\Theta$  is one of the eigenstates of  $\hat{O}_{n+1}$ . Then the probability  $P(\Theta|\Psi_n, \hat{O}_{n+1})$  that  $\Theta$  is the next state of the universe cannot factorize in any way, i.e., has only one factor. Essentially, any potential family structure is totally destroyed as far as the states are concerned, whereas from the point of view of the operators, the change appears relatively insignificant. This is the hall mark of the difference between Einstein locality and superluminal quantum correlations. Small changes consistent with the principles of relativity in operators can have consequences which reach across the universe as far as the states are concerned.

From examples such as these, we are led to believe that the mathematical differences between  $\mathcal{H}$  and  $\mathbb{H}(\mathcal{H})$  will provide important constraints on the natures of the causal sets concerned and possibly explain why one of them, based on the operators, might satisfy Einstein locality conditions whilst the other one, based on the states, need not.

It is our proposal that Einstein causality structure emerges at the point where tests factorize relative to the basic skeleton set associated with the fundamental basis for  $\mathcal{H}$ . This is based on the observation that classical cellular automata have causal structures quite analogous to light cones in relativity, provided their rules are local, that is, use information from neighbouring cells, which are defined in terms of close family relationships, such as between parents and offspring. Locality in this context is regarded as synonymous with separability, whilst non-locality is related to entanglement.

In such automata, the causal relationships between neighbouring cells can propagate signals along what effectively plays the role of light cones. In our scenario, it would be the pattern of such behaviour in the operators which effectively generates Einstein locality and which subsequently drives analogous patterns for the state outcomes, whenever quantum correlation effects can be ignored.

## X. CONCLUDING REMARKS

In this article, our aim of trying to understand the origin of causal set structure in the universe has rested on two hypotheses: one, that the universe is a fully self-contained quantum system, and two, that it is described in terms of a large but finite tensor product of elementary quantum subregisters, or qubits. We have found that these assumptions lead in a natural way to a picture of a quantum universe in which factorization and entanglement of states and observables can provide a basis for a causal set picture to emerge. Central to our discussion is von Neumann’s state reduction concept, which we believe is not an ugly and ad hoc blemish on the face of Schrödinger mechanics (as the many-worlds and decoherence theorists would have us believe), but a necessary concept directly relevant to what happens in the laboratory and the wider universe. It encodes quantum principles concerning the acquisition of information and underpins for example the “no-cloning” theorem. Given a state  $\Psi$ , we may attempt to extract information such as position by a number of position tests. If state reduction did not occur during each individual outcome, so that  $\Psi$  was invariant to each test of position, then we could proceed to extract momentum information (say), and end up having more information about the state than the uncertainty principle permits.

We have outlined a general framework, but it is clear that many details await further investigation. The programme of emergence, that is, the explanation of how the universe that we believe we see arises from our paradigm, is a hard problem which will take a long time to explore in any detail. It touches upon a major area of physics which has been virtually unexplored in any detail to date, that is, the description of physics from within, i.e., endo-physics. This must inevitably be the correct approach to any investigation which attempts to discuss the universe in a consistent way.

## ACKNOWLEDGMENTS

J.E. is grateful to E.P.S.R.C (UK) for a research studentship.

## APPENDIX: THE STAGES PARADIGM

In this section we review the stages paradigm [20, 27] in its general form, which is a mathematical framework describing the dynamical behaviour of a fully quantized, self contained and self-referential universe. By this is meant the extension of the standard principles of quantum mechanics to encompass the entire universe. In this paradigm the universe behaves as a quantum automaton, that is, a generalized quantum computation in a vast Hilbert space composed of many quantum subregisters.

1. Spacetime as a manifold does not exist *per se* but is an emergent concept, as are concepts of metric, dimensionality of space, reference frames and observers. Pre-emergent time, or “exo-time”, is synonymous with the quantum process of change of one stage of the universe to the next and is discrete, the origin of this temporal discreteness being quantum state reduction. It follows that successive stages may be labelled by an integer  $n$ .
2. At any given time  $n$ , the universe is in a unique stage  $\Omega_n \equiv \Omega(\Psi_n, \mathbb{I}_n, \mathcal{R}_n)$  which has three essential components:
  - i)  $\Psi_n$ , the current *state of the universe*, is an element in a Hilbert space  $\mathcal{H}$  of enormous dimension  $N \gg 1$ . The emergence of classical space and the separability of physical systems in the universe support the assumption that  $\mathcal{H}$  is a tensor product of a very large number of elementary Hilbert spaces, such as qubits. Any state of the universe is always a pure state and there are no external observers.
  - ii)  $\mathbb{I}_n$ , the current *information content* is information over and above that contained in  $\Psi_n$ , such as which test (see below)  $O_n$  produced  $\Psi_n$ .  $\mathbb{I}_n$  is needed for the dynamics and is classical in that it can be regarded as certain insofar as the dynamical rules are concerned governing the future evolution of the universe;
  - iii)  $\mathcal{R}_n$ , the current *rules*, govern the dynamical development of the universe;
3. For any given stage  $\Omega_n$ , all other stages such as  $\Omega_{n+1}, \Omega_{n-1}$  can only be discussed in terms of *conditional probabilities*, relative to the condition that the universe is in stage  $\Omega_n$ . This is a mathematical representation of the concept of *process time*;

4. The dynamics in the paradigm follows all of the standard principles of quantum mechanics [14], except for the non-existence of semi-classical observers with free will, and occurs as follows:
  - i) The current state of the universe (referred to as the *present*)  $\Psi_n$  is the unique outcome (modulo inessential phase) of some unique test  $O_n$ , represented by a strong element  $\hat{O}_n$  of  $\mathbb{H}(\mathcal{H})$ , the set of Hermitian operators on  $\mathcal{H}$ .  $\Psi_n$  acts as the initial state for the next test, represented by  $\hat{O}_{n+1}$ , which is also a strong element of  $\mathbb{H}(\mathcal{H})$ .
  - ii) As a strong operator,  $\hat{O}_{n+1}$  is associated with a unique preferred basis,  $\mathfrak{B}_{n+1}$ , which consists of the eigenstates of  $\hat{O}_{n+1}$ . These form a complete orthonormal set and the next state of the universe  $\Psi_{n+1}$  is one of these possible eigenstates.
  - iii) The factors which determine  $\hat{O}_{n+1}$  depend *only* on  $\Omega_n \equiv \Omega(\Psi_n, \mathbb{I}_n, \mathcal{R}_n)$  and are currently not understood, but do *not* involve any external observer making a free choice. Given  $\hat{O}_{n+1}$ , however, the conditional probability  $P(\Psi_{n+1} = \Phi | \Psi_n, \hat{O}_{n+1})$  that the next state of the universe  $\Psi_{n+1}$  is a particular eigenstate  $\Phi$  of  $\hat{O}_{n+1}$  is given by the standard quantum rule due to Born, i.e.

$$P(\Psi_{n+1} = \Phi | \Psi_n, \hat{O}_{n+1}) = |\langle \Phi | \Psi_n \rangle|^2,$$

assuming the vectors  $\Psi_n, \Phi$  are normalized to unity.

More generally, if we do *not* know what  $\hat{O}_{n+1}$  is, we should use the full stage-stage probability

$$P(\Omega_{n+1}^{A\alpha} | \Omega_n) = |\langle \Phi^{A\alpha} | \Psi_n \rangle|^2 P(\hat{O}_{n+1}^A | \Omega_n),$$

where  $P(\hat{O}_{n+1}^A | \Omega_n)$  is the conditional probability that  $\hat{O}_{n+1}^A$  is selected from  $\mathbb{H}(\mathcal{H})$  given  $\Omega_n$ , and  $\Phi^{A\alpha} \in \mathcal{H}$  is one of the eigenstates of  $\hat{O}_{n+1}^A$ .

These probabilities are meaningful only from the point of view of endo-physical observers (macroscopic patterns of factorization of states and observables) such as physicists who are attempting to understand what the future of the universe may be like.

5. Although stages appear to jump in a serial and absolute way, as labelled by exo-time, an important caveat to this idea involves the concept of *null test* [20], which allows for a “multi-fingered” view of time. Under some circumstances, parts of the universe generated by long gone stages

and represented by certain factors in  $\Psi_n$  may remain “frozen” for many successive jumps of the universe and change only during some future test. Such a possibility arises if the state of the universe is factorizable, which we assume here. When some of these factors remain unchanged from jump to jump, the result is effectively one where time regarded in terms of information change appears local. This internal time is called “endo-time”, and it is non-integrable, i.e., it is path-dependent, unlike exo-time. On emergent scales this should provide a dynamical origin for classical general relativity, including special relativity as a special case. In mathematical terms, this phenomenon originates in the fact that a quantum test of a state does not alter that state if the state is already an eigenstate of the test [28]. Such a test has no real physical content and leads to no change of information in any part of the universe;

6. After each jump  $\Psi_n \rightarrow \Psi_{n+1}$ , the information content  $\mathbb{I}_n$  and the rules  $\mathcal{R}_n$  are updated to  $\mathbb{I}_{n+1}$  and  $\mathcal{R}_{n+1}$  respectively and the whole process is repeated. According to R. Buecheri [29], how the rules might change must somehow be encoded into the rules themselves, i.e., they are also the “rules of the rules”.

## References

- [1] J. A. Wheeler, in *Quantum Theory and Gravitation*, (Academic Press, Inc, New York, 1980), edited by A.R. Marlow.
- [2] M. Stuckey, in *Studies of the Structure of Time: from Physics to Psycho(Patho)Loggy*, edited by R. Buccheri, V. Di Gesù and M. Saniga (Kluwer, Dordrecht, 1999), 121.
- [3] L. Bombelli, J. Lee, D. Meyer, and R.D. Sorkin, Phys. Rev. Lett **59**, 521 (1987).
- [4] G. Brightwell and R. Gregory, Phys. Rev. Lett **66**, 260 (1991).
- [5] D. P. Rideout and R.D. Sorkin, Phys. Rev D **61**, 024002 (2000).
- [6] D.P. Rideout and R.D. Sorkin, Phys. Rev D **63**, 104011 (2001).
- [7] G. Brightwell *et al*, arXiv:gr-qc/0202097 (2002), and Proceedings of Alternative Natural Philosophy Association Meeting, August 16-21, Cambridge, UK (2001)
- [8] T. Regge, Il Nuovo Cimento **19**(3), 558 (1961).
- [9] M.B. Plenio and P. L. Knight, Rev. Mod. Phys.**70**, 101 (1998)
- [10] F. Markopoulou, Class. Quant. Grav **17** 2059 (2000).
- [11] M. Requardt, arXiv:gr-qc/9902031 (1999).
- [12] A.G. Howson, *A Handbook of Terms Used in Algebra and Analysis* (Cambridge University Press, 1972).
- [13] H. Price, *Time's Arrow* (Oxford University Press, 1997).
- [14] A. Peres, *Quantum Theory: Concepts and Methods* (Kluwer Academic Publishers, 1993).
- [15] H. Fink and H. Leschke, Found. Phys. Lett **13**, 345 (2000).
- [16] T. Breuer, Phil. Science **62**, 197 (1995).
- [17] B. S. DeWitt and N. Graham, *The Many-Worlds Interpretation of Quantum Mechanics* (Princeton University Press, 1973).
- [18] D. Deutsch, *The Fabric of Reality* (The Penguin Press, 1997).



- [19] D. Deutsch, arXiv:quant-ph/0104033 (2001).
- [20] G. Jaroszkiewicz, arXiv:quant-ph/0105013 (2001).
- [21] J. Eakins and G. Jaroszkiewicz, J. Phys. A: Math. Gen. **36**, 517 (2003).
- [22] S. Kochen and E. Specker, J. Math. and Mech. **17**, 59 (1967).
- [23] V. Scarani, W. Tittel, H. Zbinden and N. Gisin, Phys. Lett. A **276**, 1 (2000)
- [24] J. D. Bjorken and S. D. Drell, *Relativistic Quantum Fields* (McGraw-Hill Inc, 1965).
- [25] S. Gasiorowicz, *Elementary Particle Physics* (John Wiley and Sons, 1967).
- [26] P. A. Zizzi, arXiv:gr-qc/0103002 (2001).
- [27] J. Eakins and G. Jaroszkiewicz, arXiv:quant-ph/0203020 (2002).
- [28] P.A.M. Dirac, *The Principles of Quantum Mechanics* (Clarendon Press, 1958).
- [29] R. Buccheri, in *The Nature of Time: Geometry, Physics and Perception* (Kluwer Dordrecht, 2003), edited by R. Buccheri, M. Saniga and M. Stuckey.